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三维颗粒增强复合材料细观温度场计算

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摘 要: 将高阶理论推广到三维情况,采用界面平均温度代替温度函数中待定系数作为未知量,并取消了亚子胞的概念,减少了求解方程数量,有效地提高了计算效率.在单胞的热分析过程中,利用热流与平均温度之间的关系建立子胞热传导方程,进行温度场求解,并与有限元法计算结果进行了比较,验证了该方法的正确性和有效性.

关 键 词: 高阶理论; 复合材料; 细观结构; 温度场

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Temperature fields calculation of 3-D particle reinforced composite

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Abstract: The higher-order theory was developed to three-dimension, coefficients of temperature function were replaced by surface-averaged temperature to be primary variables of equations, and the concept of sub-cells was canceled. The number of equations was reduced so that computational efficiency was increased effectively. During the thermal analysis of represent volume element, heat exchange equations which were established by the relationship between hear flux and average temperature were used to compute temperature, The comparison of numerical results with finite element analysis demonstrates the correctness and validity of this method.

Key words: higher-order theory; composite; micro-structure; temperature fields

金属基或陶瓷基复合材料由于组分材料的热物性参数不同,导致材料在制备和使用过程中产生较大的热应力.因此,准确地预测复合材料细观温度场,并计算温度载荷下的残余应力和热应力具有非常重要的实际意义^[1-2].

在复合材料增强相形状比较复杂的细观温度场预测中,有限元法的精度较高但比较耗时;传统的细观模型计算效率高,但要求夹杂数目很大、夹杂相特征尺寸明显小于宏观尺寸、温度梯度与夹杂相比不是很大^[1];Aboudi 等人^[3-6]提

出了一种宏-细观结合的高阶细观力学模型,克服了传统细观力学模型的缺点,且与有限元法的计算结果吻合较好. Bansal^[7]改进了高阶理论,用界面平均量代替温度函数系数作为未知量,有效地减少了求解方程数量,减少了计算时间,孙志刚等人^[8-10]也开展了相应的研究工作.但上述研究都局限于二维问题,本文将改进的高阶理论扩展到三维,对颗粒增强复合材料细观温度场进行了预测,并与有限元计算结果进行了对比.

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1 改进的高阶理论

将周期性复合材料的代表性体元 (RVE) 划分为 $N_\alpha \times N_\beta \times N_\gamma$ 个子胞, D , H 和 L 分别为代表性体元在 x_1 , x_2 和 x_3 方向的长度, 如图 1 所示, 子胞在三个方向的尺寸分别为 d_α , h_β 和 l_γ , 即: 在正六面体子胞内, 则有

$$D = \sum_{\alpha=1}^{N_\alpha} d_\alpha; H = \sum_{\beta=1}^{N_\beta} h_\beta; L = \sum_{\gamma=1}^{N_\gamma} l_\gamma$$

假设子胞 (α, β, γ) 的温度 $T^{(\alpha, \beta, \gamma)}$ 为局部坐标 $\bar{x}_1^{(\alpha)}$, $\bar{x}_2^{(\beta)}$ 和 $\bar{x}_3^{(\gamma)}$ 的二阶多项式

$$\begin{aligned} T^{(\alpha, \beta, \gamma)} = & T_{(000)}^{(\alpha, \beta, \gamma)} + \bar{x}_1^{(\alpha)} T_{(100)}^{(\alpha, \beta, \gamma)} + \bar{x}_2^{(\beta)} T_{(010)}^{(\alpha, \beta, \gamma)} + \\ & \bar{x}_3^{(\gamma)} T_{(001)}^{(\alpha, \beta, \gamma)} + \frac{1}{2} \left[3 \bar{x}_1^{(\alpha)^2} - \frac{d_\alpha^2}{4} \right] T_{(200)}^{(\alpha, \beta, \gamma)} + \\ & \frac{1}{2} \left[3 \bar{x}_2^{(\beta)^2} - \frac{h_\beta^2}{4} \right] T_{(020)}^{(\alpha, \beta, \gamma)} + \frac{1}{2} \left[3 \bar{x}_3^{(\gamma)^2} - \frac{l_\gamma^2}{4} \right] T_{(002)}^{(\alpha, \beta, \gamma)} \end{aligned} \quad (1)$$

其中 $T_{(ijk)}^{(\alpha, \beta, \gamma)}$ 为待定系数, 局部坐标系的原点为子胞的几何中心, 各坐标轴与总体坐标轴平行.

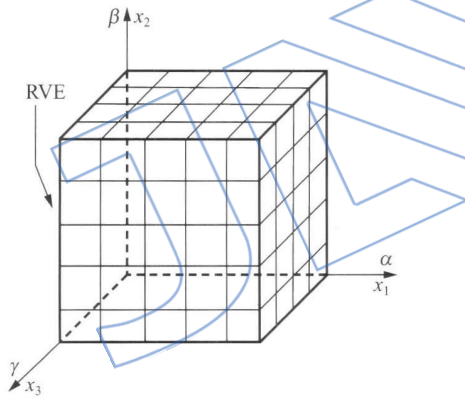


图 1 $N_\alpha \times N_\beta \times N_\gamma$ 规模单元

Fig. 1 Discretization of the repeating unit cell

子胞 (α, β, γ) 内的热传导 Fourier 定律为

$$q_i^{(\alpha, \beta, \gamma)} = -\lambda_i^{(\alpha, \beta, \gamma)} \frac{\partial T^{(\alpha, \beta, \gamma)}}{\partial x_i^{(\cdot)}} \quad (2)$$

其中 $i = 1, 2, 3$; $q_i^{(\alpha, \beta, \gamma)}$ 为热流密度; $\lambda_i^{(\alpha, \beta, \gamma)}$ 为导热系数, 当 $\lambda_i^{(\alpha, \beta, \gamma)}$ 在各方向取不同值时, 为各向异性材料; $x_i^{(\cdot)}$ 表示局部坐标.

将假设的温度场代入 Fourier 定律得

$$\begin{aligned} q_1^{(\alpha, \beta, \gamma)} = & -\lambda_1^{(\alpha, \beta, \gamma)} \left[T_{(100)}^{(\alpha, \beta, \gamma)} + 3 \bar{x}_1^{(\alpha)} T_{(200)}^{(\alpha, \beta, \gamma)} \right] \\ q_2^{(\alpha, \beta, \gamma)} = & -\lambda_2^{(\alpha, \beta, \gamma)} \left[T_{(010)}^{(\alpha, \beta, \gamma)} + 3 \bar{x}_2^{(\beta)} T_{(020)}^{(\alpha, \beta, \gamma)} \right] \\ q_3^{(\alpha, \beta, \gamma)} = & -\lambda_3^{(\alpha, \beta, \gamma)} \left[T_{(001)}^{(\alpha, \beta, \gamma)} + 3 \bar{x}_3^{(\gamma)} T_{(002)}^{(\alpha, \beta, \gamma)} \right] \end{aligned} \quad (3)$$

为了提高计算的效率, 采用表面平均量代替温度函数中的待定系数作为未知量, 为了得到反映子胞平均温度和平均热流之间关系的细观热传导矩阵, 需要建立表面平均热流、表面平均温度与细观变量 $T_{(ijk)}^{(\alpha, \beta, \gamma)}$ 之间的关系.

定义子胞 x_2 负方向的表面平均热流

$$\begin{aligned} \bar{Q}_2^{(\alpha, \beta, \gamma)} = & \frac{1}{d_\alpha l_\gamma} \int_{-d_\alpha/2}^{d_\alpha/2} \int_{-l_\gamma/2}^{l_\gamma/2} q_2^{(\alpha, \beta, \gamma)} \left[\bar{x}_1^{(\alpha)}, -\frac{h_\beta}{2}, \bar{x}_3^{(\gamma)} \right] d\bar{x}_3^{(\gamma)} d\bar{x}_1^{(\alpha)} \end{aligned} \quad (4)$$

同理可得到子胞其它表面的平均热流表达式, 将得到的热流方程代入式 (4), 积分并写成矩阵形式

$$\begin{aligned} \begin{bmatrix} \bar{Q}_1^+ \\ \bar{Q}_1^- \end{bmatrix}^{(\alpha, \beta, \gamma)} = & -\lambda_1^{(\alpha, \beta, \gamma)} \begin{bmatrix} 1 & \frac{3d_\alpha}{2} \\ 1 & -\frac{3d_\alpha}{2} \end{bmatrix} \begin{bmatrix} T_{(100)} \\ T_{(200)} \end{bmatrix}^{(\alpha, \beta, \gamma)} \\ \begin{bmatrix} \bar{Q}_2^+ \\ \bar{Q}_2^- \end{bmatrix}^{(\alpha, \beta, \gamma)} = & -\lambda_2^{(\alpha, \beta, \gamma)} \begin{bmatrix} 1 & \frac{3h_\beta}{2} \\ 1 & -\frac{3h_\beta}{2} \end{bmatrix} \begin{bmatrix} T_{(010)} \\ T_{(020)} \end{bmatrix}^{(\alpha, \beta, \gamma)} \\ \begin{bmatrix} \bar{Q}_3^+ \\ \bar{Q}_3^- \end{bmatrix}^{(\alpha, \beta, \gamma)} = & -\lambda_3^{(\alpha, \beta, \gamma)} \begin{bmatrix} 1 & \frac{3l_\gamma}{2} \\ 1 & -\frac{3l_\gamma}{2} \end{bmatrix} \begin{bmatrix} T_{(001)} \\ T_{(002)} \end{bmatrix}^{(\alpha, \beta, \gamma)} \end{aligned} \quad (5)$$

定义子胞 x_2 负方向表面平均温度

$$\begin{aligned} \bar{T}_2^{(\alpha, \beta, \gamma)} = & \frac{1}{d_\alpha l_\gamma} \int_{-d_\alpha/2}^{d_\alpha/2} \int_{-l_\gamma/2}^{l_\gamma/2} T^{(\alpha, \beta, \gamma)} \left[\bar{x}_1^{(\alpha)}, -\frac{h_\beta}{2}, \bar{x}_3^{(\gamma)} \right] d\bar{x}_3^{(\gamma)} d\bar{x}_1^{(\alpha)} \end{aligned} \quad (6)$$

同理可得子胞其他表面的平均温度表达式, 将假设的子胞温度场代入并转化矩阵形式为

$$\begin{aligned} \begin{bmatrix} T_{(100)} \\ T_{(200)} \end{bmatrix}^{(\alpha, \beta, \gamma)} = & \begin{bmatrix} \frac{1}{d_\alpha} & -\frac{1}{d_\alpha} \\ \frac{2}{d_\alpha^2} & \frac{2}{d_\alpha^2} \end{bmatrix} \begin{bmatrix} \bar{T}_1^+ \\ \bar{T}_1^- \end{bmatrix}^{(\alpha, \beta, \gamma)} - \frac{4}{d_\alpha^2} \begin{bmatrix} 0 \\ T_{(000)} \end{bmatrix}^{(\alpha, \beta, \gamma)} \\ \begin{bmatrix} T_{(010)} \\ T_{(020)} \end{bmatrix}^{(\alpha, \beta, \gamma)} = & \begin{bmatrix} \frac{1}{h_\beta} & -\frac{1}{h_\beta} \\ \frac{2}{h_\beta^2} & \frac{2}{h_\beta^2} \end{bmatrix} \begin{bmatrix} \bar{T}_2^+ \\ \bar{T}_2^- \end{bmatrix}^{(\alpha, \beta, \gamma)} - \frac{4}{h_\beta^2} \begin{bmatrix} 0 \\ T_{(000)} \end{bmatrix}^{(\alpha, \beta, \gamma)} \end{aligned}$$

$$\begin{bmatrix} T_{(001)} \\ T_{(002)} \end{bmatrix}^{(\alpha, \beta, \gamma)} = \begin{bmatrix} \frac{1}{l_y} & -\frac{1}{l_y} \\ \frac{2}{l_y^2} & \frac{2}{l_y^2} \end{bmatrix} \begin{bmatrix} \bar{T}_3^+ \\ \bar{T}_3^- \end{bmatrix}^{(\alpha, \beta, \gamma)} - \frac{4}{l_y^2} \begin{bmatrix} 0 \\ T_{(000)} \end{bmatrix}^{(\alpha, \beta, \gamma)} \quad (7)$$

子胞热流的稳态热传导方程为

$$\frac{\partial q_1^{(\alpha, \beta, \gamma)}}{\partial x_1^{(\alpha)}} + \frac{\partial q_2^{(\alpha, \beta, \gamma)}}{\partial x_2^{(\beta)}} + \frac{\partial q_3^{(\alpha, \beta, \gamma)}}{\partial x_3^{(\gamma)}} = 0 \quad (8)$$

将热流方程(3)代入式(8), 进行体积平均并化简, 得

$$[\lambda_a T_{(200)} + \lambda_b T_{(020)} + \lambda_c T_{(002)}]^{(\alpha, \beta, \gamma)} = 0 \quad (9)$$

再将二阶细观变量式(7)代入式(9), 则有

$$\begin{aligned} T_{(000)}^{(\alpha, \beta, \gamma)} &= \frac{\lambda_1^{(\alpha, \beta, \gamma)}}{2 \bar{\lambda}_1^{(\alpha, \beta, \gamma)}} (\bar{T}_1^+ + \bar{T}_1^-)^{(\alpha, \beta, \gamma)} + \\ &\frac{\lambda_2^{(\alpha, \beta, \gamma)}}{2 \bar{\lambda}_2^{(\alpha, \beta, \gamma)}} (\bar{T}_2^+ + \bar{T}_2^-)^{(\alpha, \beta, \gamma)} + \\ &\frac{\lambda_3^{(\alpha, \beta, \gamma)}}{2 \bar{\lambda}_3^{(\alpha, \beta, \gamma)}} (\bar{T}_3^+ + \bar{T}_3^-)^{(\alpha, \beta, \gamma)} \end{aligned} \quad (10)$$

其中

$$\begin{aligned} \bar{\lambda}_1^{(\alpha, \beta, \gamma)} &= \lambda_1^{(\alpha, \beta, \gamma)} + \frac{d_a^2}{h_\beta^2} \lambda_2^{(\alpha, \beta, \gamma)} + \frac{d_a^2}{l_\gamma^2} \lambda_3^{(\alpha, \beta, \gamma)}; \\ \bar{\lambda}_2^{(\alpha, \beta, \gamma)} &= \frac{h_\beta^2}{d_a^2} \lambda_1^{(\alpha, \beta, \gamma)} + \lambda_2^{(\alpha, \beta, \gamma)} + \frac{h_\beta^2}{l_\gamma^2} \lambda_3^{(\alpha, \beta, \gamma)}; \\ \bar{\lambda}_3^{(\alpha, \beta, \gamma)} &= \frac{l_\gamma^2}{d_a^2} \lambda_1^{(\alpha, \beta, \gamma)} + \frac{l_\gamma^2}{h_\beta^2} \lambda_2^{(\alpha, \beta, \gamma)} + \lambda_3^{(\alpha, \beta, \gamma)}. \end{aligned}$$

将式(10)代入式(7), 得到表面平均温度表示的子胞温度函数, 将其代入式(5)得到子胞细观热传导矩阵

$$\begin{bmatrix} \bar{Q}_1^+ \\ -\bar{Q}_1^- \\ \bar{Q}_2^+ \\ -\bar{Q}_2^- \\ \bar{Q}_3^+ \\ -\bar{Q}_3^- \end{bmatrix}^{(\alpha, \beta, \gamma)} =$$

$$\begin{bmatrix} \kappa_1 & \kappa_2 & \kappa_3 & \kappa_4 & \kappa_5 & \kappa_6 \\ \kappa_1 & \kappa_2 & \kappa_3 & \kappa_4 & \kappa_5 & \kappa_6 \\ \kappa_1 & \kappa_2 & \kappa_3 & \kappa_4 & \kappa_5 & \kappa_6 \\ \kappa_1 & \kappa_2 & \kappa_3 & \kappa_4 & \kappa_5 & \kappa_6 \\ \kappa_1 & \kappa_2 & \kappa_3 & \kappa_4 & \kappa_5 & \kappa_6 \\ \kappa_1 & \kappa_2 & \kappa_3 & \kappa_4 & \kappa_5 & \kappa_6 \end{bmatrix}^{(\alpha, \beta, \gamma)} \begin{bmatrix} \bar{T}_1^+ \\ \bar{T}_1^- \\ \bar{T}_2^+ \\ \bar{T}_2^- \\ \bar{T}_3^+ \\ \bar{T}_3^- \end{bmatrix}^{(\alpha, \beta, \gamma)} \quad (11)$$

其中 κ_{ij} 为材料的热物性参数和子胞细观尺寸的表达式, $i, j = 1, 2, \dots, 6$.

相邻子胞间的界面温度在平均意义上是连续的, 则有

$$\bar{T}_1^+ (\alpha, \beta, \gamma) = \bar{T}_1^- (\alpha+1, \beta, \gamma) = \bar{T}_1^{\alpha+1, \beta, \gamma}$$

其中 $\alpha = 1, \dots, N_\alpha - 1$; $\beta = 1, \dots, N_\beta$; $\gamma = 1, \dots, N_\gamma$.

$$\bar{T}_2^+ (\alpha, \beta, \gamma) = \bar{T}_2^- (\alpha, \beta+1, \gamma) = \bar{T}_2^{\alpha, \beta+1, \gamma}$$

其中 $\alpha = 1, \dots, N_\alpha$; $\beta = 1, \dots, N_\beta - 1$; $\gamma = 1, \dots, N_\gamma$.

$$\bar{T}_3^+ (\alpha, \beta, \gamma) = \bar{T}_3^- (\alpha, \beta, \gamma+1) = \bar{T}_3^{\alpha, \beta, \gamma+1}$$

其中 $\alpha = 1, \dots, N_\alpha$; $\beta = 1, \dots, N_\beta$; $\gamma = 1, \dots, N_\gamma - 1$.

由给定的热力学边界条件确定复合材料外边界表面温度 $\bar{T}_1^{(1, \beta, \gamma)}$, $\bar{T}_1^{(N_\alpha+1, \beta, \gamma)}$, $\bar{T}_2^{(\alpha, 1, \gamma)}$, $\bar{T}_2^{(\alpha, N_\beta+1, \gamma)}$, $\bar{T}_3^{(\alpha, \beta, 1)}$ 和 $\bar{T}_3^{(\alpha, \beta, N_\gamma+1)}$, 待子胞界面平均温度数量为 $(N_\alpha - 1) N_\beta N_\gamma + N_\alpha (N_\beta - 1) N_\gamma + N_\alpha N_\beta (N_\gamma - 1)$ 个。

相邻子胞间的界面热流在平均意义上也是连续的, 则有

$$\bar{Q}_1^+ (\alpha, \beta, \gamma) - \bar{Q}_1^- (\alpha+1, \beta, \gamma) = 0$$

其中 $\alpha = 1, \dots, N_\alpha - 1$; $\beta = 1, \dots, N_\beta$; $\gamma = 1, \dots, N_\gamma$.

$$\bar{Q}_2^+ (\alpha, \beta, \gamma) - \bar{Q}_2^- (\alpha, \beta+1, \gamma) = 0$$

其中 $\alpha = 1, \dots, N_\alpha$; $\beta = 1, \dots, N_\beta - 1$; $\gamma = 1, \dots, N_\gamma$.

$$\bar{Q}_3^+ (\alpha, \beta, \gamma) - \bar{Q}_3^- (\alpha, \beta, \gamma+1) = 0$$

其中 $\alpha = 1, \dots, N_\alpha$; $\beta = 1, \dots, N_\beta$; $\gamma = 1, \dots, N_\gamma - 1$.

再由细观热传导矩阵得到关于表面平均温度和边界平均温度的方程

$$\begin{aligned} &\kappa_{12}^{(\alpha, \beta, \gamma)} \bar{T}_1^{(\alpha, \beta, \gamma)} + (\kappa_{11}^{(\alpha, \beta, \gamma)} + \kappa_{22}^{(\alpha+1, \beta, \gamma)}) \bar{T}_1^{(\alpha+1, \beta, \gamma)} + \kappa_{21}^{(\alpha+1, \beta, \gamma)} \bar{T}_1^{(\alpha+2, \beta, \gamma)} + \kappa_{44}^{(\alpha, \beta, \gamma)} \bar{T}_2^{(\alpha, \beta, \gamma)} + \\ &\kappa_{33}^{(\alpha, \beta, \gamma)} \bar{T}_2^{(\alpha, \beta+1, \gamma)} + \kappa_{24}^{(\alpha+1, \beta, \gamma)} \bar{T}_2^{(\alpha+1, \beta, \gamma)} + \kappa_{23}^{(\alpha+1, \beta, \gamma)} \bar{T}_2^{(\alpha+1, \beta+1, \gamma)} + \kappa_{66}^{(\alpha, \beta, \gamma)} \bar{T}_3^{(\alpha, \beta, \gamma)} + \\ &\kappa_{55}^{(\alpha, \beta, \gamma)} \bar{T}_3^{(\alpha, \beta, \gamma+1)} + \kappa_{26}^{(\alpha+1, \beta, \gamma)} \bar{T}_3^{(\alpha+1, \beta, \gamma)} + \kappa_{54}^{(\alpha+1, \beta, \gamma)} \bar{T}_3^{(\alpha+1, \beta, \gamma+1)} = 0 \\ &\kappa_{32}^{(\alpha, \beta, \gamma)} \bar{T}_1^{(\alpha, \beta, \gamma)} + \kappa_{31}^{(\alpha, \beta, \gamma)} \bar{T}_1^{(\alpha+1, \beta, \gamma)} + \kappa_{42}^{(\alpha, \beta+1, \gamma)} \bar{T}_1^{(\alpha, \beta+1, \gamma)} + \kappa_{41}^{(\alpha, \beta+1, \gamma)} \bar{T}_1^{(\alpha+1, \beta+1, \gamma)} + \kappa_{34}^{(\alpha, \beta, \gamma)} \bar{T}_2^{(\alpha, \beta, \gamma)} + \\ &(\kappa_{33}^{(\alpha, \beta, \gamma)} + \kappa_{44}^{(\alpha, \beta+1, \gamma)}) \bar{T}_2^{(\alpha, \beta+1, \gamma)} + \kappa_{43}^{(\alpha, \beta+1, \gamma)} \bar{T}_2^{(\alpha, \beta+2, \gamma)} + \kappa_{66}^{(\alpha, \beta, \gamma)} \bar{T}_3^{(\alpha, \beta, \gamma)} + \\ &\kappa_{55}^{(\alpha, \beta, \gamma)} \bar{T}_3^{(\alpha, \beta, \gamma+1)} + \kappa_{46}^{(\alpha, \beta+1, \gamma)} \bar{T}_3^{(\alpha, \beta+1, \gamma)} + \kappa_{45}^{(\alpha, \beta+1, \gamma)} \bar{T}_3^{(\alpha, \beta+1, \gamma+1)} = 0 \end{aligned}$$

$$\begin{aligned} &K_{62}^{(a,\beta,\gamma)}\overline{T_1^{(a,\beta,\gamma)}} + K_{61}^{(a,\beta,\gamma)}\overline{T_1^{(a+1,\beta,\gamma)}} + K_{62}^{(a,\beta,\gamma+1)}\overline{T_1^{(a,\beta,\gamma+1)}} + K_{61}^{(a,\beta,\gamma+1)}\overline{T_1^{(a+1,\beta,\gamma+1)}} + \\ &K_{64}^{(a,\beta,\gamma)}\overline{T_2^{(a,\beta,\gamma)}} + K_{63}^{(a,\beta,\gamma)}\overline{T_2^{(a,\beta+1,\gamma)}} + K_{64}^{(a,\beta,\gamma+1)}\overline{T_2^{(a,\beta,\gamma+1)}} + K_{63}^{(a,\beta,\gamma+1)}\overline{T_2^{(a,\beta+1,\gamma+1)}} + \\ &K_{66}^{(a,\beta,\gamma)}\overline{T_3^{(a,\beta,\gamma)}} + (K_{65}^{(a,\beta,\gamma)} + K_{66}^{(a,\beta,\gamma+1)})\overline{T_3^{(a,\beta,\gamma+1)}} + K_{65}^{(a,\beta,\gamma+1)}\overline{T_3^{(a,\beta,\gamma+2)}} = 0 \end{aligned}$$

(12)

式 (12) 中共有 $(Na - 1) N\beta N\gamma + Na(N\beta - 1) N\gamma + NaN\beta(N\gamma - 1)$ 个方程,但复合材料温度或热流边界的确定需要 $2(N\beta N\gamma + NaN\gamma + NaN\beta)$ 个方程,为了求解上述方程组成的方程组,避免矩阵奇异,在边界处至少要定义一个 $\overline{T_1^{(\cdot)}}$, $\overline{T_2^{(\cdot)}}$ 和 $\overline{T_3^{(\cdot)}}$.如果只有热流边界,则求解的温度分布可能不止一个.这与力学分析是相同的,当边界处只定义应力条件而没有位移边界条件时,可能产生刚体位移.

将这 $(Na + 1) N\beta N\gamma + Na(N\beta + 1) N\gamma + NaN\beta(N\gamma + 1)$ 个关于界面平均温度方程写成矩阵形式为

$$K\overline{T} = \overline{Q}$$

(13)

其中 $\overline{T}$ 为平均温度矩阵,包括单胞边界和子胞界面平均温度,形式为

$$\overline{T} = [\overline{T_1^{(11)}}, \dots, \overline{T_1^{(N\beta 1)}}, \dots, \overline{T_1^{(1 N\gamma)}}, \dots, \overline{T_1^{(N\beta N\gamma)}},$$
$$\overline{T_2^{(11)}}, \dots, \overline{T_2^{(N\beta 1)}}, \dots, \overline{T_2^{(1 N\gamma)}}, \dots, \overline{T_2^{(N\beta N\gamma)}},$$
$$\overline{T_3^{(11)}}, \dots, \overline{T_3^{(N\beta 1)}}, \dots, \overline{T_3^{(1 N\gamma)}}, \dots, \overline{T_3^{(N\beta N\gamma)}}]$$

其中

$$\overline{T_1^{(\beta\gamma)}} = [\overline{T_1^{(1,\beta,\gamma)}}, \dots, \overline{T_1^{(Na+1,\beta,\gamma)}}]$$
$$\overline{T_2^{(a\gamma)}} = [\overline{T_2^{(a,1,\gamma)}}, \dots, \overline{T_2^{(a,N\beta+1,\gamma)}}]$$
$$\overline{T_3^{(a\beta)}} = [\overline{T_3^{(a,\beta,1)}}, \dots, \overline{T_3^{(a,\beta,N\gamma+1)}}]$$

矩阵 $\overline{Q}$ 由单胞边界的热流组成,具体形式略. K 为宏观热传导矩阵,由子胞的热物性参数和细观结构尺寸构成,并可以表示为

$$K = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}$$

其中 K_{11} , K_{22} , K_{33} 分别与各自方向量有关的矩阵,且为对角阵.其余子矩阵分别是各个方向耦合矩阵,且为稀疏矩阵.宏观热传导矩阵 K 为 $(Na + 1) N\beta N\gamma + Na(N\beta + 1) N\gamma + NaN\beta(N\gamma + 1)$ 阶方阵.其中

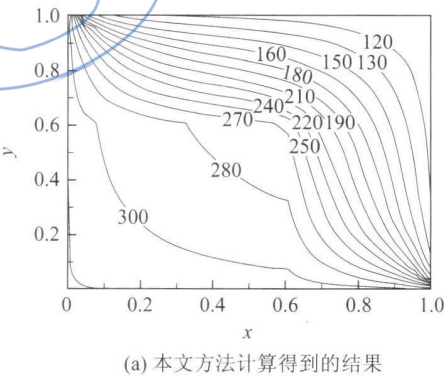
- K_{11} 为 $(Na + 1) N\beta N\gamma$ 阶方阵;
- K_{22} 为 $Na(N\beta + 1) N\gamma$ 阶方阵;
- K_{33} 为 $NaN\beta(N\gamma + 1)$ 阶方阵;

- K_{12} 为 $(Na + 1) N\beta N\gamma \times Na(N\beta + 1) N\gamma$ 阶矩阵;
- K_{13} 为 $(Na + 1) N\beta N\gamma \times NaN\beta(N\gamma + 1)$ 阶方阵;
- K_{21} 为 $Na(N\beta + 1) N\gamma \times (Na + 1) N\beta N\gamma$ 阶矩阵;
- K_{23} 为 $Na(N\beta + 1) N\gamma \times NaN\beta(N\gamma + 1)$ 阶方阵;
- K_{31} 为 $NaN\beta(N\gamma + 1) \times (Na + 1) N\beta N\gamma$ 阶方阵;
- K_{32} 为 $NaN\beta(N\gamma + 1) \times Na(N\beta + 1) N\gamma$ 阶方阵.

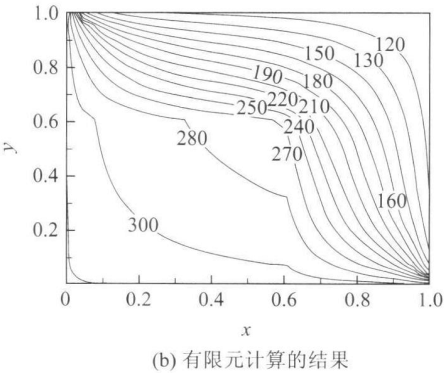
2 算例分析

2.1 二维长方体纤维增强复合材料温度场预测

纤维形状为长方形,其中一个方向纤维与代表性体元的长度相同,该方向的两个表面为绝热平面,平面图及组分材料的热物性参数见文献 [8-9],热分析边界条件:上边和右边的温度为 $100\text{ }^{\circ}\text{C}$,下边和右边的温度为 $300\text{ }^{\circ}\text{C}$,计算得到温度等值线图,并与有限元法的计算结果进行比较,如图 2 所示.



(a) 本文方法计算得到的结果



(b) 有限元计算的结果

图 2 方形纤维的温度场分布 (单位: $^{\circ}\text{C}$)

Fig. 2 Temperature fields with square fiber (unit: $^{\circ}\text{C}$)

2.2 包含立方体颗粒相的三维温度场计算

取各向同性代表性体元边长为 1 的立方体,计算时离散为 $10 \times 10 \times 10$ 的等分子胞,其中心有一个 $6 \times 6 \times 6$ 的纤维,如图 3 所示。

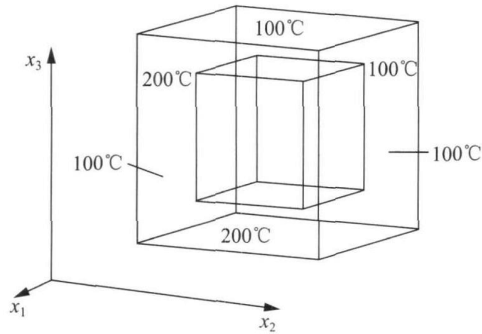
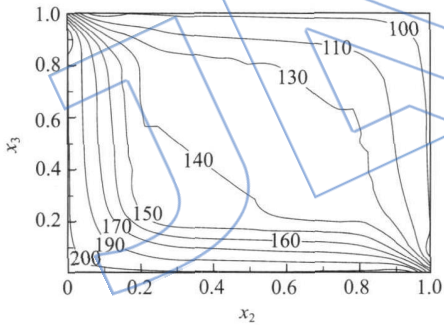


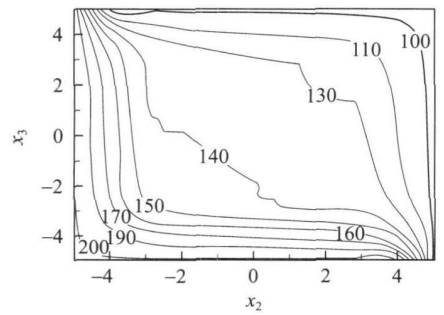
图 3 包含立方体纤维的三维示意图

Fig. 3 Three-dimensional composite of cube

增强相与基体的导热系数之比为 14/ 150,取 x_2 方向与 x_3 方向的负面均为 200 °C,其余表面均为 100 °C,计算结果如图 4、图 5 所示。



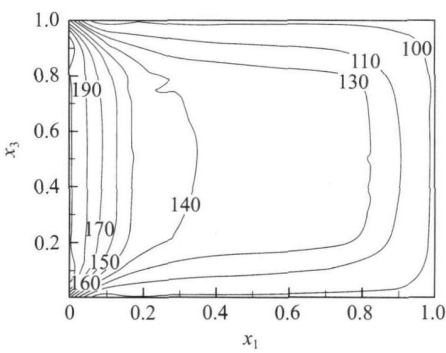
(a) 本文方法计算得到的结果



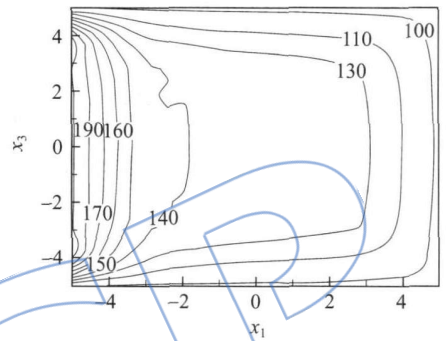
(b) 有限元计算的结果

图 4 $x_1 = 0.5$ 截面温度分布的对比图 (单位: °C)

Fig. 4 Temperature fields at $x_1 = 0.5$ cross section (unit: °C)



(a) 本文方法计算得到的结果



(b) 有限元计算的结果

图 5 $x_2 = 0.5$ 截面温度分布的对比图 (单位: °C)

Fig. 5 Temperature fields at $x_2 = 0.5$ cross section (unit: °C)

3 结 论

复合材料细观温度场计算是分析复合材料残余应力及热应力的重要步骤,本文采用改进的高阶理论推导了复合材料温度场分布公式,对二维纤维增强和颗粒增强复合材料的稳态温度场进行了预测,主要结论为:

- 1) 对高阶理论方法进行了改进,并推广到三维分析中,使求解方程数目减少了约 55 %,有效地降低了求解时间;
- 2) 消除了原始算法中亚子胞的概念,使分网进一步简化;
- 3) 对复合材料细观稳态温度场进行了预测,与有限元解具有较好的一致性。

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